Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Discrete Mathematics

Subject Code: 4TE04DSM1 **Branch: B.Tech (CE)**

Semester: 4 **Date: 20/10/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Find the least and greatest element in the poset ({1, 2, 3, 4, 6, 12}, D) if they exist. **(02)**

b) Define: Equivalence relation, Sub algebra. **(02)**

c) State Pigeonhole principle. **(02)**

d) Find the atom and anti-atom of $\langle P(X), \subseteq \rangle$, if X is finite set. **(02)**

e) Prove that if $a = b$ then $ab' + a'b = 0$. **(01)**

f) Define: tree and simple graph. **(02)**

g) $(Z_7, +_7)$ is a cyclic group.- True or False? **(01)**

h) Define: Complement of Fuzzy set. **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions. **(14)**

a) Let $\langle L, \leq \rangle$ be a lattice $a, b \in L$ then prove that **(07)**

$i)\ a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b \quad ii)\ a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

b) For a lattice $\langle S_{30}, D \rangle$, answer the following questions: **(07)**

i) Find cover of each element and draw the Hasse diagram.
ii) Find lower bound, upper bound, greatest lower bound, least upper bound of $A = \{2, 6\}$.
iii) Find the least and greatest element of it.

Q-3 Attempt all questions **(14)**

a) Show that $\{1, 5, 7, 11\}$ is a subgroup of $(Z_{12}^*, \times_{12})$, where $\times_{12}$ is multiplication modulo 12. **(05)**

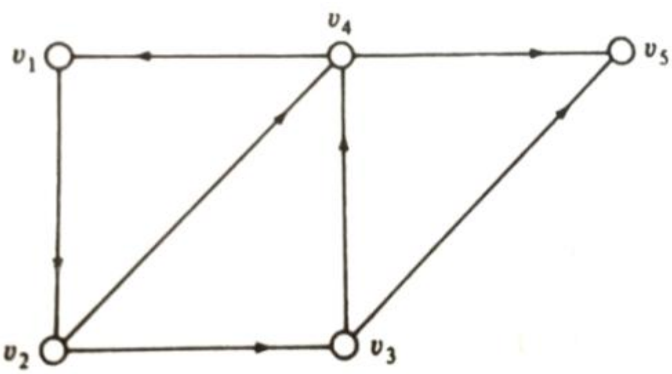
b) Prove that $\langle P(X), \subseteq \rangle$ is a complemented lattice and also draw the Hasse diagram of it, where $X = \{a, b, c\}$. **(05)**

c)	Obtain the sum of product canonical form of the Boolean expression in three variables $\alpha(x,y,z) = x \oplus (y * z')$.	**(04)**

Q-4	**Attempt all questions**	**(14)**
a)	Let $\langle L, \le \rangle$ be a lattice and $a,b,c \in L$ then show that the following are equivalent. i) $a*(b\oplus c) = (a*b)\oplus(a*c)$ ii) $a\oplus(b*c) = (a\oplus b)*(a\oplus c)$	**(07)**
b)	Let $E = \{a,b,c\}$, $\underset{\sim}{A} = \{(a,0.3),(b,0.8),(c,0.5)\}$, $\underset{\sim}{B} = \{(a,0.7),(b,0.6),(c,0.4)\}$ then find the following: 1) $\underset{\sim}{A} \cup \underset{\sim}{B}$ 2) $\underset{\sim}{A} \cdot \underset{\sim}{B}$ 3) $\underset{\sim}{A} \hat{+} \underset{\sim}{B}$ 4) $\underset{\sim}{A} - \underset{\sim}{B}$ 5) $\underset{\sim}{A} \cap \underset{\sim}{B}$ 6) $\underset{\sim}{A}'$ 7) $\underset{\sim}{B}'$	**(07)**

Q-5	**Attempt all questions**	**(14)**
a)	State and prove Stone's representation theorem.	**(10)**
b)	State De Morgan's law for fuzzy subsets and prove any one.	**(04)**

Q-6	**Attempt all questions**	**(14)**
a)	i) Draw the graph represented by given adjacency matrix $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$.	**(05)**
	ii) Write the adjacency matrix from the given digraph for the order v_1, v_2, v_3, v_4 and v_5	



b)	State and prove Lagrange's theorem.	**(05)**
c)	Prove that $\langle S_{20}, D \rangle$ is a lattice.	**(04)**

Q-7	**Attempt all questions.**	**(14)**
a)	Define: unilaterally connected graph, cycle, reachable set, node base, level of vertex.	**(05)**
b)	Prove that $(Z_6, +_6)$ is a group. Is it commutative?	**(05)**
c)	By using mathematical induction prove that $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.	**(04)**

Q-8 Attempt all questions. **(14)**

a) Define a complete binary tree and draw a directed tree from following and also find the representation of binary tree. $\left(v_0\left(v_1\left(v_2\right)\left(v_3\left(v_4\right)\left(v_5\right)\right)\right)\left(v_6\left(v_7\left(v_8\right)\right)\left(v_9\right)\left(v_{10}\right)\right)\right)$. **(07)**

b) Show that the set Q \ {-1} is an abelian group with respect to the binary operation $a*b=a+b+ab$, for all a, b ∈ G. **(07)**